B12T5 1. Schulaufgabe am 18. 11. 10

1.0 $f_k(x) = \frac{1}{12}(x^3 - 2kx^2 + 12x)$; $k \in \mathbb{R}_0^+$

1.1. Für $k = 0$: $f_0(x) = \frac{1}{12}(x^3 + 12x)$ nur unger. Exp. : P-Sym z. Urspr.

Für $k \neq 0$: keine besondere Sym.

1.2 $f_k'(x) = \frac{1}{12}(3x^2 - 4kx + 12) = 0$

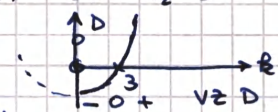
$D = (4k)^2 - 4 \cdot 3 \cdot 12 = 16k^2 - 144 = 16(k^2 - 9) = 16(k+3)(k-3) = 0$

$(k_1 = -3)$ $k_2 = 3$

Für $0 \leq k < 3$: $D < 0$ und keine hor. Tang.

Für $k = 3$: $D = 0$, hor. Tang. bei TEP.

Für $k > 3$: $D > 0$: 2 hor. Tang. bei Extremalstellen



1.3 $f_k(k) = \frac{1}{12}(k^3 - 2k \cdot k^2 + 12k) = \frac{1}{12}(12k - k^3) = y_0$; $x_0 = k$

$f_k'(k) = \frac{1}{12}(3 \cdot k^2 - 4k^2 + 12) = \frac{1}{12}(-k^2 + 12) = m$

$b = y_0 - m x_0 = \frac{1}{12}(12k - k^3) - \frac{1}{12}(12 - k^2) \cdot k$

$= \frac{1}{12}(12k - k^3) - \frac{1}{12}(12k - k^3) = 0$

$t_k(x) = \frac{1}{12}(12 - k^2)\,x = x - \frac{1}{12}k^2 x$; Büschel durch Ursprung

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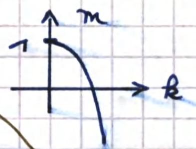
$m = (1 - \frac{1}{12}k^2) \Rightarrow m_{max} = 1$ für $k = 0$:



1.4 $d_k(x) = \frac{1}{12}(x^3 - 2kx^2 + 12x) - \frac{1}{12}(12 - k^2)\,x$

$= \frac{1}{12}(x^3 - 2kx^2 + 12x - 12x + k^2 x)$

$= \frac{1}{12}(x^3 - 2kx^2 + k^2 x) = \frac{1}{12}x(x^2 - 2kx + k^2)$

$= \frac{1}{12}x(x - k)^2 \rightarrow x_1 = 0$ einf. ; $x_2 = k$ do.

$d_k(x) = f_k(x) - t_k(x) = 0 \Leftrightarrow f_k(x) = t_k(x)$

: Die NST sind die Abszissen der SP (1-f) oder Berührpkte (do.)

1.5 $13x + 12y + 11 = 0 \Leftrightarrow y = -\frac{13}{12}x - \frac{11}{12}$; $m = -\frac{13}{12}$

$m_{tk} = \frac{1}{12}(12 - k^2) = -\frac{13}{12} \Leftrightarrow 12 - k^2 = -13 \Leftrightarrow k^2 = 25$

$\Rightarrow k_1 = 5$; $(k_2 = -5 \notin \mathbb{R}_0^+)$

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2.0 $a = 5$; $f_5(x) = f(x) = \frac{1}{12}\left(x^3 - 10x^2 + 12x\right)$

2.1 • $f(x) = \frac{1}{12}x\left(x^2 - 10x + 12\right)$; $x_1 = 0$

: $x_{2/3} = \dfrac{10 \pm \sqrt{100 - 4 \cdot 12}}{2} = 5 \pm \sqrt{13}$ $\qquad x_2 = 5 + \sqrt{13} \approx 8{,}61$; $x_3 = 5 - \sqrt{13} \approx 1{,}39$

• $f(x) = \frac{1}{12}x\left(x - 5 - \sqrt{13}\right)\left(x - 5 + \sqrt{13}\right)$

2.2 $f'(x) = \frac{1}{12}\left(3x^2 - 20x + 12\right) = 0$

: $x_{1/2} = \frac{1}{6}\left(20 \pm \sqrt{400 - 4 \cdot 3 \cdot 12}\right) = \frac{1}{6}(20 \pm 16)$

• $x_1 = 4/6 = 2/3$; $f\left(\frac{2}{3}\right) = \frac{26}{81}$ $(\approx 0{,}32)$

$x_2 = 6$ $\qquad f(6) = -6$

x		$2/3$		6	
VZ $f'(x)$	+	0	−	0	+
$f(x)$	sms	HOP	smf	TIP	sms

: • HOP $\left(\frac{2}{3} \middle| \frac{26}{81}\right)$

TIP $(6 \mid -6)$

2.3 $t_5(x) = -\frac{13}{12}x$; HOP ; TIP ; $N_{1/2/3}$; damit G_f u. G_t . . . . .

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3 : $f(x) = x^2 - 3$; $f'(x) = 2x$ $\qquad \left(x^2 - 3 = 0 \Leftrightarrow x = \underset{(-)}{+}\sqrt{3}\right)$

$x_{n+1} = x_n - \dfrac{x_n^2 - 3}{2x_n}$

n	x_n	$f(x_n)$	$f'(x_n)$
0	2	1	4
• 1	$7/4$	$1/16$	$7/2$
• 2	$97/56$	$\frac{1}{3136}$	$97/28$
• 3	$1{,}732 \approx \sqrt{3}$		

• Prozentuale Abw. : $\dfrac{1{,}732 - \sqrt{3}}{\sqrt{3}}$

$\approx 0{,}00000024\%$

BE		NP
52 …	50	15
…	47,5	14
…	44,5	13
	42	12
	39,5	11
	37	10
	34,5	9
	31,5	8
	29	7
	26,5	6
	24	5
	21,5	4
	17,5	3
	14	2
	10,5	1
10 …	0	0

Aufgabe 2.3

$f(x) = \frac{1}{12}(x^3 - 10x^2 + 12x)$; $t_5(x) = -\frac{13}{12}x$